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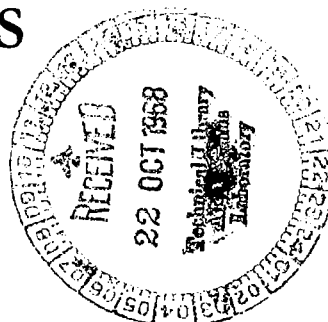
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SIMPLIFIED TECHNIQUE FOR DETERMINING PERICYNTION ALTITUDE OF LUNAR ORBITS

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Langley Research Center

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SIMPLIFIED TECHNIQUE FOR DETERMINING PERICYNTHION ALTITUDE OF LUNAR ORBITS

By L. Keith Barker
Langley Research Center

SUMMARY

A study has been made of a simplified technique for determining the pericynthion altitude of transfer orbits from a selenocentric circular orbit at 80 nautical miles (148 160 m) to a lower altitude. The transfer orbits had pericynthion altitudes ranging from 0 to 100 000 feet (30 480 m) and radial injection velocities between ± 10 ft/sec (± 3 m/sec). The technique consisted of measuring the angle between a reference vehicle in circular orbit and the lunar horizon at two specified times. Pericynthion altitude was then determined graphically from these measurements. The effects of errors in measuring the angles, in determining the sighting times, in the circular reference orbit, and in horizon uncertainties were considered. It appears that the technique would be useful for predicting the pericynthion altitude of the transfer orbits and, if necessary, for providing information for orbit corrections.

INTRODUCTION

In the Apollo mission, the lunar module is injected into an elliptical transfer orbit from the circular orbit of the command module to a lower specified pericynthion altitude. At this point, thrust is applied and the lunar module descends to a soft landing on the lunar surface. The primary guidance during the initial lunar missions is scheduled to be automatic; however, since moderate errors in the transfer maneuver could result in an unsafe orbit (impact or near-impact trajectory), mission reliability and crew safety could be enhanced if simplified methods were developed to determine the pericynthion altitude. (See refs. 1 and 2.)

The purpose of this study is to examine the feasibility of using the line-of-sight angle between an orbiting reference vehicle in circular orbit and the lunar horizon, as measured from a descending transfer vehicle, to determine the pericynthion altitude of the transfer orbit. A wide range of off-nominal transfer orbits is considered, pericynthion altitudes ranging from 0 to 100 000 feet (30 480 m). The effects of errors in sighting angle, sighting time, circular reference orbit, and horizon irregularities are considered. All motion is planar.

SYMBOLS

The units for the physical quantities used in this paper are given in both U.S. Customary Units and the International System of Units (SI). (See ref. 3.)

r	radial distance from center of moon, feet (meters)
r_m	radius of moon, 5 702 000 feet (1738 kilometers)
g_m	gravitational acceleration at surface of moon, 5.32 feet/second ² (1.62 meters/second ²)
h	altitude above lunar surface, feet (meters)
θ	angular travel over lunar surface from transfer-orbit injection, degrees or radians
t	time from transfer-orbit injection, seconds
K	angle between reference vehicle in circular orbit and horizon, measured from transfer vehicle, degrees
ω	angular rate of orbiting reference vehicle, degrees/second or radians/second

Subscripts:

1	conditions at first visual sighting
2	conditions at second visual sighting
p	conditions at pericyynthion
o	nominal conditions at transfer-orbit injection
s	refers to orbiting reference vehicle

Dots over symbols indicate derivatives with respect to time. A Δ preceding a parameter indicates a change in that parameter from a nominal value.

STATEMENT OF PROBLEM

It is assumed that a transfer vehicle has separated from a reference vehicle in a selenocentric circular orbit and is descending on a transfer orbit to a lower altitude for subsequent landing. The purpose of this study is to examine a simple manual technique which requires a minimum of instrumentation for determining the pericyynthion altitude of the transfer orbit. The technique examined consists of measuring the angle between the orbiting reference vehicle and the lunar horizon at two specified times, as shown in figure 1, and then determining the pericynthion altitude of the transfer orbit directly from these angular measurements.

The attractive features of this technique are:

- (1) It is simple and requires only an optical device and a time reference
- (2) It does not require any knowledge of the local vertical
- (3) It is independent of any particular surface feature or orbit injection point.

ANALYSIS

The approach used in this paper was to compute a series of transfer orbits having different pericynthion altitudes, and then to examine the sighting angles at specific times. Appreciable differences in the angles for the various trajectories would indicate that the sighting angles could be used to distinguish the orbits and, therefore, afford a means of predicting the pericynthion altitude.

Equations of Motion

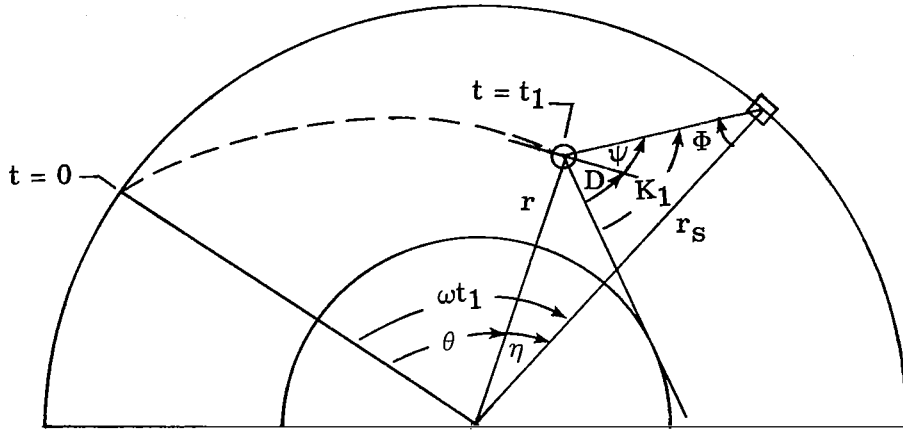
A digital computer was used to compute the location of the transfer vehicle at the specified sighting times from the following equations:

$$\ddot{r} - r\dot{\theta}^2 + g_m \left(\frac{r_m}{r} \right)^2 = 0 \quad (1)$$

$$2\dot{\theta}\dot{r} + r\ddot{\theta} = 0 \quad (2)$$

These equations describe planar movement of a point mass near a spherical homogenous moon.

The following sketch shows the geometry involved in deriving the auxiliary equations used to compute the first sighting angle K_1 measured at t_1 relative to the horizon in the direction of motion:



The auxiliary equations are:

$$\eta = \omega t_1 - \theta \quad (3)$$

$$\Phi = \tan^{-1} \left(\frac{\sin \eta}{\frac{r_s}{r} - \cos \eta} \right) \quad (4)$$

$$D = \cos^{-1} \frac{r_m}{r} \quad (5)$$

$$\psi = 90 - \Phi - \eta \quad (6)$$

$$K_1 = D + \psi \quad (7)$$

The second sighting angle K_2 , measured at t_2 relative to the horizon opposite the direction of motion (fig. 1), can be computed by similar equations. This convention of measuring K_1 relative to the horizon in the direction of motion and K_2 relative to the horizon opposite the direction of motion was chosen because of the smaller angles involved.

Transfer Orbits

The transfer orbits considered in this investigation are orbits from a circular orbit at an altitude of 80 nautical miles (148 160 m) to a lower altitude. The nominal transfer orbit selected was a Hohmann orbit with a pericyynthion altitude of 50 000 feet (15 240 mm).

In order to establish this nominal transfer orbit, the circular velocity of the transfer vehicle was decreased by 97.4 ft/sec (29.7 m/sec). The nominal polar velocity components after instantaneous injection were $\dot{r}_0 = 0$ and $(r\dot{\theta})_0 = 5189.5$ ft/sec (1581.8 m/sec). Off-nominal transfer orbits were generated by varying these velocity components over a range of approximately ± 10 ft/sec (± 3 m/sec).

Selection of Sighting Times

Preliminary calculations indicated that the simplified prediction technique is less sensitive to errors if the first angular measurement is made soon after injection and the second angular measurement is made as late as possible. This trend was also noted in reference 1. The sighting times, however, should be chosen so that the first sighting time allows ample time for the astronaut to prepare for the measurement, and the second sighting time allows sufficient time after the measurement to estimate the pericynthion altitude and then to prepare for any corrective action that is required. Ten minutes after orbit injection was selected as the time to make the first angular measurement. The time selected for the second angular measurement was 45 minutes after injection. This latter time was selected so that all transfer orbits were at altitudes above 50 000 feet (15 240 m) at the time of the second sighting.

RESULTS AND DISCUSSION

Determination of Pericynthion Altitude and Velocity

Figure 2 shows the variation of pericynthion altitude as a function of injection velocity errors $\Delta(r\dot{\theta})_0$ and $\Delta\dot{r}_0$. As indicated, the pericynthion altitude is rather insensitive to errors of up to ± 10 ft/sec (± 3 m/sec) in \dot{r}_0 , the effect being less than 300 feet (91 m) change in pericynthion altitude. However, errors in $(r\dot{\theta})_0$ change pericynthion altitude by about 4300 feet (1311 m) per 1 ft/sec (0.3 m/sec) change in $(r\dot{\theta})_0$. (See fig. 2.)

It was found that errors in \dot{r}_0 and $(r\dot{\theta})_0$ caused changes in the angles K_1 and K_2 , measured at the nominal times of 10 minutes and 45 minutes after separation. Figure 3 is a plot of the results and shows sighting angles K_1 and K_2 as a function of $\Delta\dot{r}_0$ and $\Delta(r\dot{\theta})_0$. The curves were generated by fixing $\Delta(r\dot{\theta})_0$, computing trajectories for several values of $\Delta\dot{r}_0$, and observing the K_1 and K_2 angles at the specified times. The curves of figure 3 can be used to relate the angular measurements to variation in injection conditions. Figure 2 relates variations in injection condition to pericynthion altitude. The results of figures 2 and 3, therefore, can be combined, as in figure 4, to relate pericynthion altitude to the angles K_1 and K_2 . The curves of figure 4 are nearly linear, and an equation relating pericynthion altitude to the angles K_1 and K_2 is given in feet by

$$h_p = -1\,120\,316.0 + 6596.6 K_1 + 17\,238.6 K_2 \quad (8a)$$

or in meters by

$$h_p = -341\,472.3 + 2010.6 K_1 + 5254.3 K_2 \quad (8b)$$

This equation is accurate to about ± 3000 feet (± 914 m) for the range of variables used in the study.

The results of this study also showed that the following relationship was accurate to within ± 0.3 ft/sec (± 0.1 m/sec):

$$\Delta(r\dot{\theta})_p = -3.07 \Delta(r\dot{\theta})_o \quad (9)$$

Therefore, the curves of figures 2 and 4, in addition to equation (9), provide a means of defining conditions at pericyynthion in terms of angular measurements K_1 and K_2 .

The sun angles presently being considered for the initial manned lunar landing vary from 7° to 20° above the nominal landing site, which is located approximately 192° from the injection point. Under these conditions, both horizons are illuminated at the nominal sighting times. The visibility of the reference vehicle depends upon how close the line of sight to the reference vehicle passes to the sun. For sun angles of 7° and 20° , it was found that the minimum angular location of the sun from the line of sight to the reference vehicle was 19° and 33° , respectively. No attempt was made to define acceptable sun locations stringently since the quantitative effects of glare are not fully known. (See ref. 4.)

Error Analysis

It appears that angular measurements between the line of sight to a reference vehicle in a circular orbit and that to the lunar horizon could permit accurate determination of pericynthion altitude. The results of figure 4 indicate that for the range of variables investigated in this study, the maximum sensitivities are approximately:

$$\frac{\partial h_p}{\partial K_1} = 9000 \text{ ft/deg} = 2743 \text{ m/deg} \quad (10)$$

$$\frac{\partial h_p}{\partial K_2} = 23\,000 \text{ ft/deg} = 7010 \text{ m/deg} \quad (11)$$

The maximum error in predicted pericynthion altitude, therefore, would be

$$E(h_p) = 9000 E(K_1) + 23\,000 E(K_2) \quad (12)$$

where E denotes the error of the quantity in parenthesis.

It was of interest to determine the effect of various errors on the accuracy of predicting pericyynthion altitude. Errors considered were those associated with:

- (1) Measurement of line-of-sight angles
- (2) Time at which measurements are made
- (3) Uncertainty in reference vehicle orbit
- (4) Irregularities of lunar surface

Measurement of line-of-sight angles.- This error is an error in the sighting angle from its true value and results from uncertainty in the sighting instrument or from the astronaut's use of the instrument. Such errors could be reduced by making a number of sightings in the vicinity of the selected sighting times t_1 and t_2 and then graphically obtaining the most likely values of K_1 and K_2 . If it is assumed that a high quality sextant can be used to determine K_1 and K_2 to within a probable error of $\pm 0.05^\circ$ (ref. 5), then the maximum absolute error in the predicted pericynthion altitude, as given by equation (12), is 1600 feet (488 m).

Time at which measurements are made.- By making additional calculations it was found that, for a given transfer orbit, sighting angle varies linearly with sighting time within ± 50 seconds of the nominal sighting time. The maximum sensitivities for all the transfer orbits of figure 4 are approximately:

$$\frac{\partial K_1}{\partial t_1} = 0.06 \text{ deg/sec} \quad (13)$$

$$\frac{\partial K_2}{\partial t_2} = 0.04 \text{ deg/sec} \quad (14)$$

If a consistent error of 1 second is made at each angular reading used to determine K_1 and K_2 , the maximum errors in K_1 and K_2 would then be $E(K_1) = 0.06^\circ$ and $E(K_2) = 0.04^\circ$. Substituting these errors into equation (12) gives a maximum error in the predicted pericynthion altitude of 1460 feet (445 m).

Uncertainty in reference vehicle orbit.- Suppose the orbit of the reference vehicle is confined between uncertainty limits of ± 1 nautical mile (± 1852 m) of the nominal reference orbit altitude of 80 nautical miles (148 160 m). Elliptical orbits are included. If the angles K_1 and K_2 are measured at the nominal times and figure 4 is used to estimate the pericynthion altitude, calculations show that the maximum absolute error in the predicted pericynthion altitude is approximately 5000 feet (1524 m). Linear extrapolations were required in some cases to allow for the nominal velocity injection errors.

Irregularities of lunar surface.- The curves of figure 4 are applicable to a spherical moon. It was of interest, therefore, to determine the effects of surface irregularities

(or difference in lunar radius from the assumed value) on pericyynthion altitude prediction. Computation indicated an approximately linear relationship of 1-foot (0.3-m) error predicted in pericynthion altitude for each 1-foot (0.3-m) uncertainty in lunar radius at the horizon. The maximum error in the predicted pericynthion altitude for a combination of the errors considered can be examined by using superposition of the individual sensitivities.

CONCLUDING REMARKS

A study has been made of a simplified technique for determining the pericynthion altitude of transfer orbits from a selenocentric circular orbit at 80 nautical miles (148 160 meters) to a lower altitude. The transfer orbits had pericynthion altitudes ranging from 0 to 100 000 feet (30 480 m) and radial injection velocities between ± 10 feet/second (± 3 meters/second). The technique consisted of measuring the angle between a reference vehicle in circular orbit and the lunar horizon at two specified times. Pericynthion altitude was then determined graphically from these measurements. The effect of errors in measuring the angles, in determining the sighting times, in the circular reference orbit, and in horizon uncertainties were considered. It appears that the technique would be useful for predicting the pericynthion altitude of the transfer orbits and, if necessary, to provide information for orbit corrections.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 2, 1968,
127-51-06-01-23.

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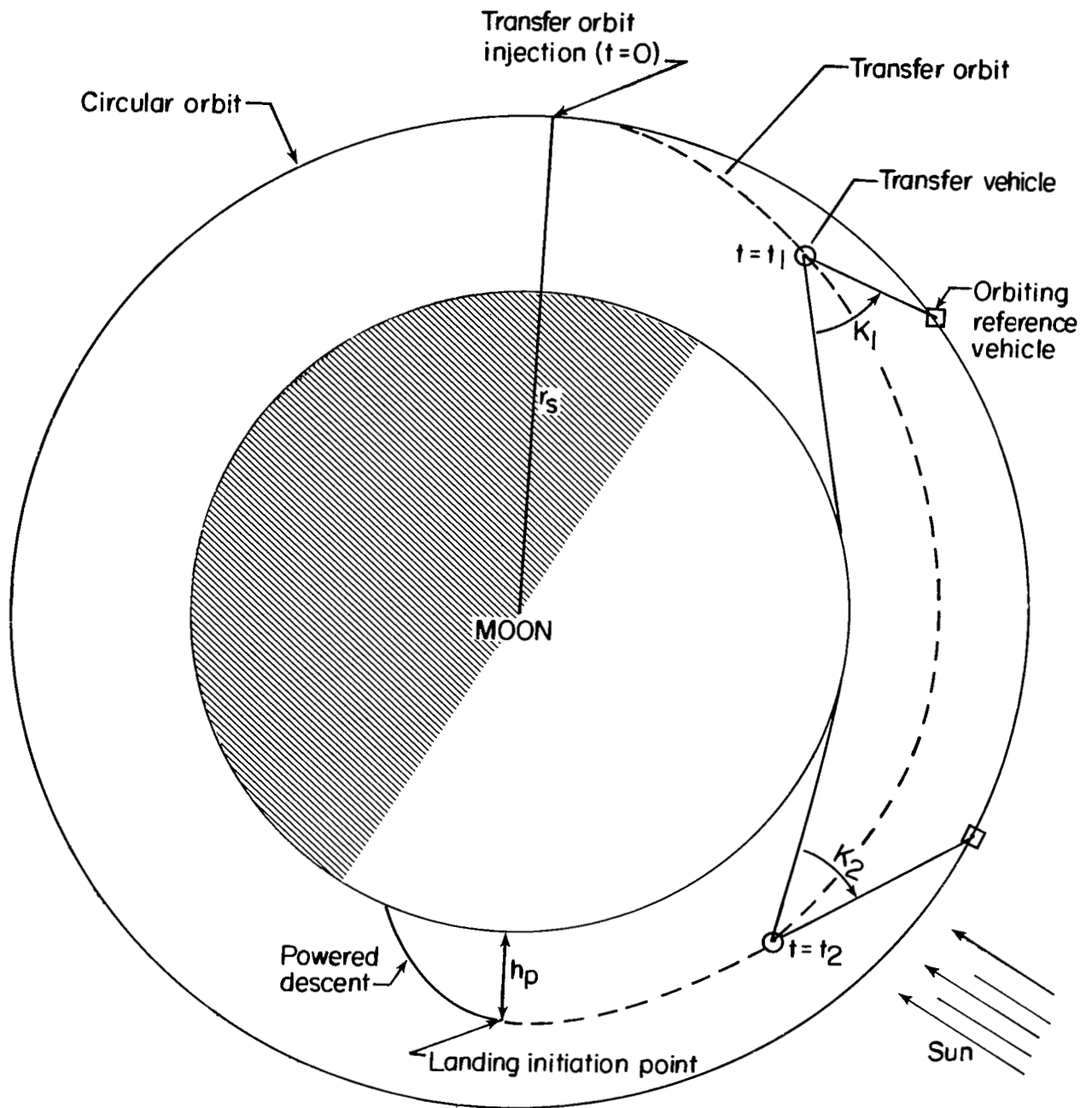


Figure 1.- Illustration of simplified technique.

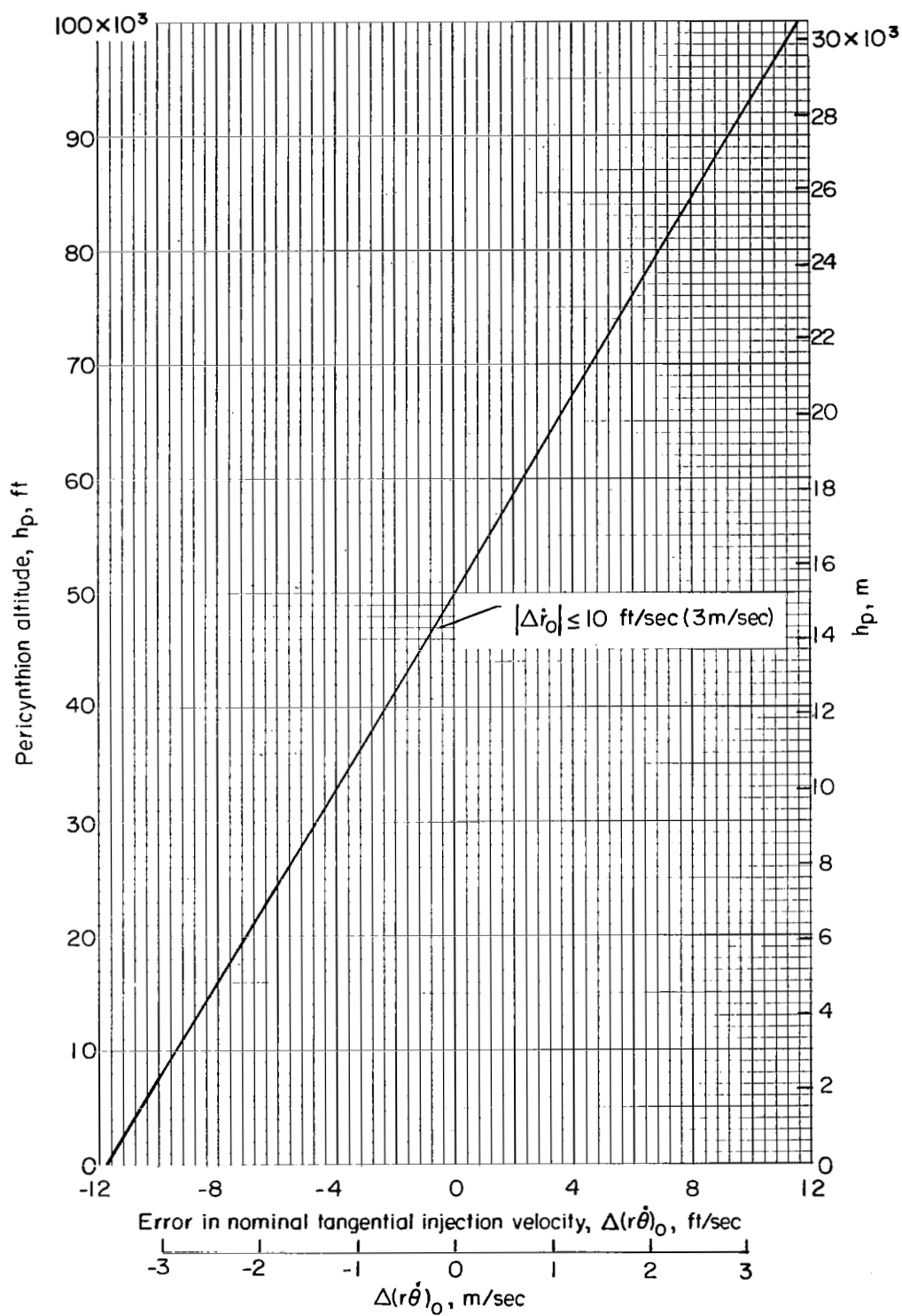


Figure 2.- Pericynthion altitude as a function of injection velocity errors.

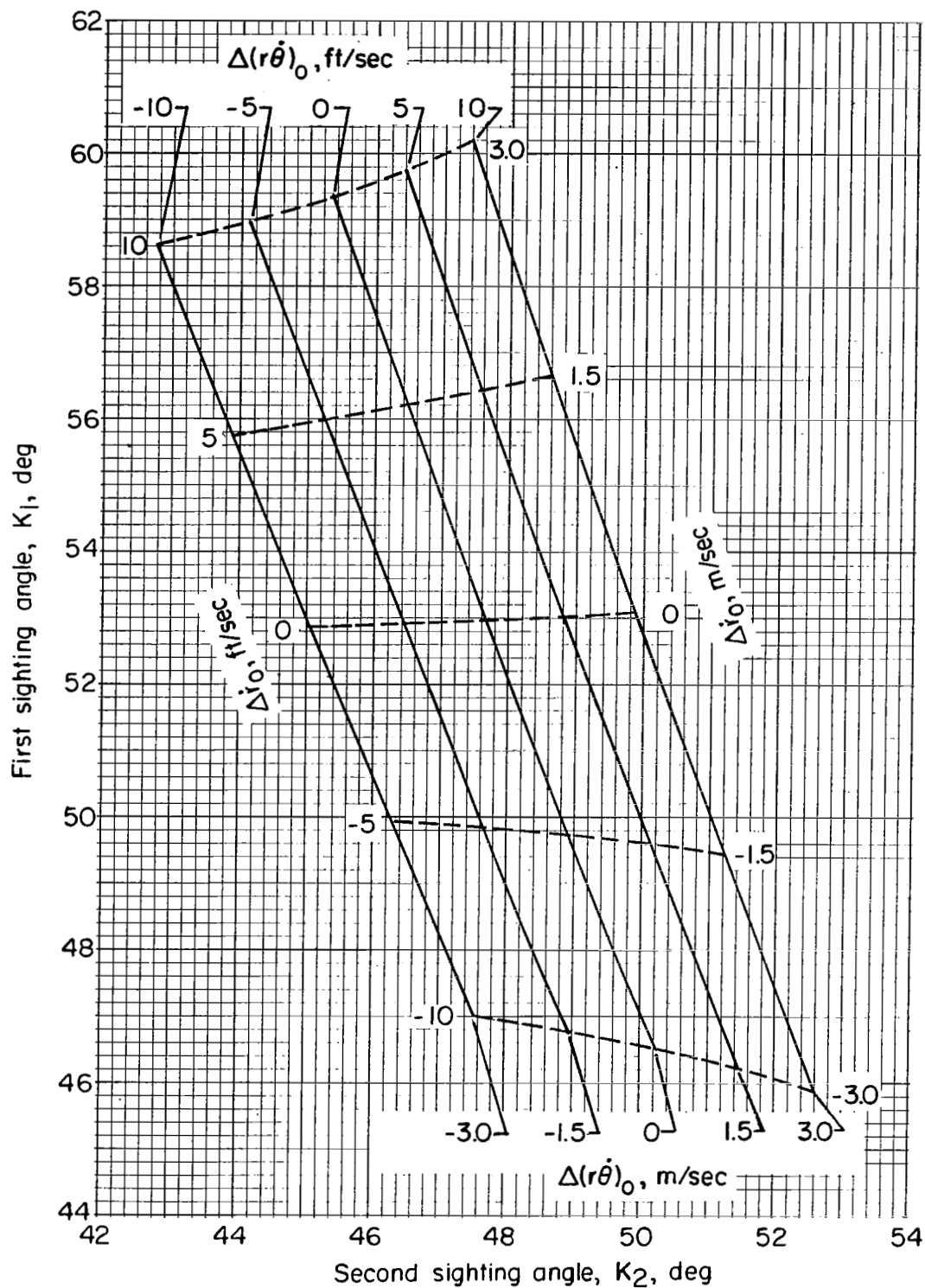


Figure 3.- Graphical relationships between the injection velocity errors and sighting angles.

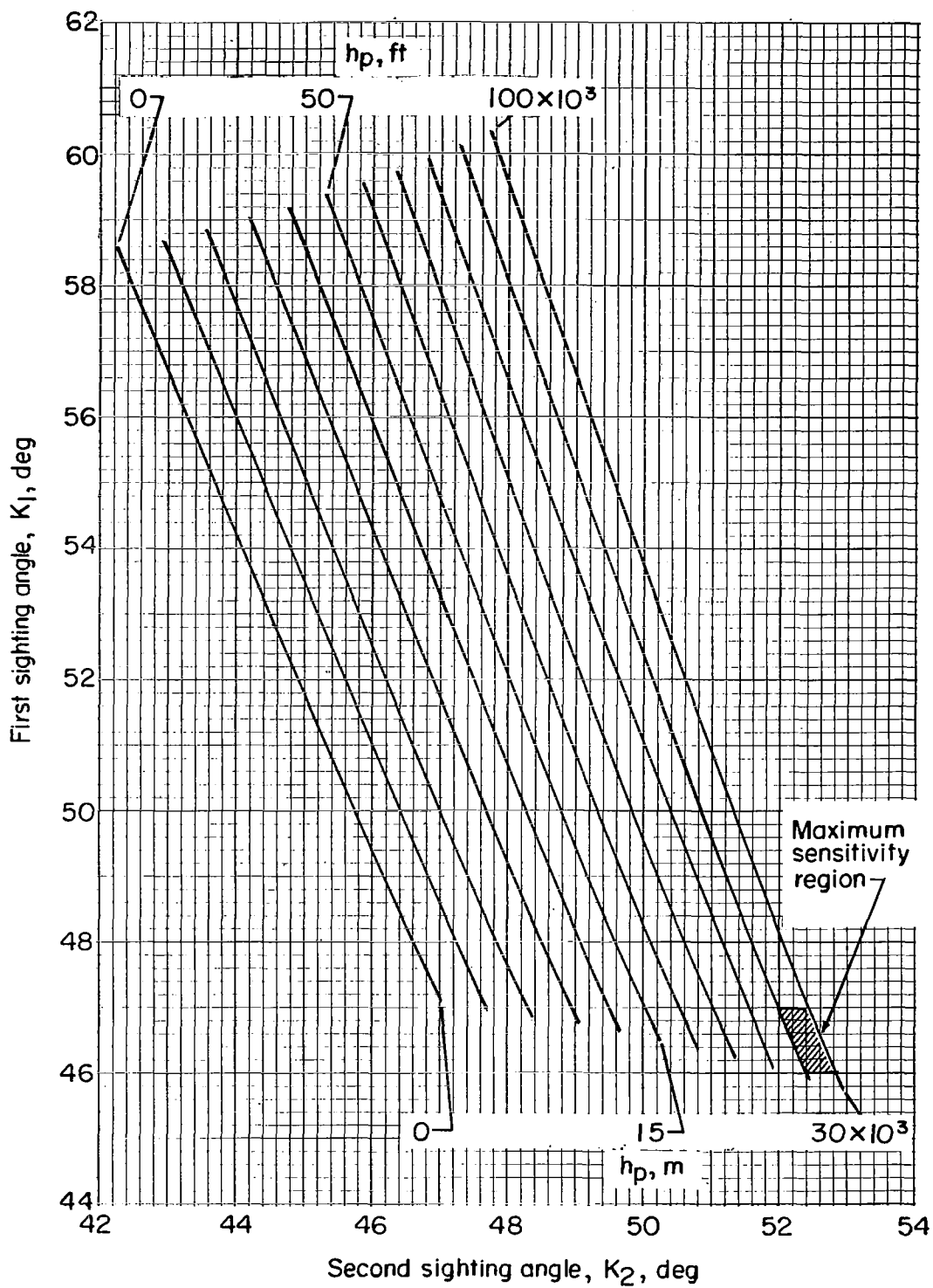


Figure 4.- Pericynthion altitude as a function of sighting angles.